Minkowski's inequality for integrals

The following inequality is a generalization of Minkowski's inequality C12.4 to double integrals. In some sense it is also a theorem on the change of the order of iterated integrals, but equality is only obtained if $p = 1$.

13.14 Theorem (Minkowski's inequality for integrals) *Let* $(X, \mathcal{A}, \mu)$ *and* $(Y, \mathcal{B}, \mu)$ *be* σ-*finite measure spaces and* $u : X \times Y \to \bar{\mathbb{R}}$ *be* $\mathcal{A} \otimes \mathcal{B}$-*measurable. Then*

$$\left(\int_X \left(\int_Y |u(x,y)| \, \nu(dy) \right)^p \mu(dx) \right)^{1/p} \leqslant \int_Y \left(\int_X |u(x,y)|^p \mu(dx) \right)^{1/p} \nu(dy)$$

holds for all $p \in [1, \infty)$, *with equality for* $p = 1$.

Proof If $p = 1$, the assertion follows directly from Tonelli's theorem 13.8. If $p > 1$ we set

$$U_k(x) := \left(\int_Y |u(x,y)| \, \nu(dy) \wedge k \right) \mathbf{1}_{A_k}(x)$$

where $A_k \in \mathcal{A}$ is a sequence with $A_k \uparrow X$ and $\mu(A_k) < \infty$. Without loss of generality we may assume that $U_k(x) > 0$ on a set of positive μ-measure, otherwise the left-hand side of the above inequality would be 0 (using Beppo Levi's theorem 9.6) and there would be nothing to prove. By Tonelli's theorem and Hölder's inequality T12.2 with $\frac{1}{p} + \frac{1}{q} = 1$ or $q = \frac{p}{p-1}$, we find

$$\int_X U_k^p(x) \, \mu(dx) \leqslant \int_X U_k^{p-1}(x) \left(\int_Y |u(x,y)| \, \nu(dy) \right) \mu(dx)$$

$$= \int_Y \int_X U_k^{p-1}(x) |u(x,y)| \, \mu(dx) \, \nu(dy)$$

$$\leqslant \int_Y \left(\int_X U_k^p(x) \mu(dx) \right)^{1-1/p} \left(\int_X |u(x,y)|^p \mu(dx) \right)^{1/p} \nu(dy).$$

The claim follows upon dividing both sides by $\left(\int_X U_k^p(x) \mu(dx) \right)^{1-1/p}$ and letting $k \to \infty$ with Beppo Levi's theorem 9.6. ∎

Problems

13.1. Prove the rules (13.2) for Cartesian products.

13.2. Let $(X, \mathcal{A}, \mu)$ and $(Y, \mathcal{B}, \nu)$ be two σ-finite measure spaces. Show that $A \times N$, where $A \in \mathcal{A}$ and $N \in \mathcal{B}$, $\nu(N) = 0$, is a $\mu \times \nu$-null set.

Measures Integrals And Martingales

Yasushi Ishikawa

Measures Integrals And Martingales:

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