

# Long time dynamics and blow-up for the focusing inhomogeneous nonlinear Schrödinger equation with spatially growing nonlinearity

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## ABSTRACT

We investigate the Cauchy problem for the focusing inhomogeneous nonlinear Schrödinger equation  $i\partial_t u + \Delta u = -|x|^b |u|^{p-1} u$  in the radial Sobolev space  $H^1_b(\mathbb{R}^N)$ , where  $b > 0$  and  $p > 1$ . We show the global existence and energy scattering in the intercritical regime, i.e.,  $p > \frac{N+1+2b}{N-2}$  and  $p < \frac{N+1+2b}{N-2}$  if  $N \geq 3$ . We also obtain blowing-up solutions for the mass-critical and mass-supercritical nonlinearities. The main difficulty, coming from the spatial growing nonlinearity, is overcome by refined Gagliardo–Nirenberg-type inequalities. Our proofs are based on improved Gagliardo–Nirenberg inequalities, the Morawetz–Sobolev approach of Dodson and Murphy [Proc. Am. Math. Soc. **145**(11), 4859–4867 (2017)], radial Sobolev embeddings, and localized virial estimates.

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## 1. INTRODUCTION

In this paper, we consider the Cauchy problem for the following focusing inhomogeneous nonlinear Schrödinger equation:

$$i\partial_t u + \Delta u = -|x|^b |u|^{p-1} u, \quad (t, x) \in \mathbb{R}_+ \times \mathbb{R}^N, \quad (1.1)$$

where  $b > 0$  and  $p > 1$ . Equation (1.1) is a special case of a more general inhomogeneous nonlinear Schrödinger equation given by

$$i\partial_t u + \Delta u = K(x) |u|^{p-1} u, \quad (1.2)$$

which arises in various physical contexts such as the propagation of a laser beam and plasma waves. Here,  $u$  is the electric field in laser optics and  $K$  is proportional to the electric density.<sup>1,2</sup> For  $p = 3$ , Eq. (1.1) can be viewed as a model of dilute Bose–Einstein condensate when the two-body interactions of the condensate are considered.

The Cauchy problem for (1.2) was first investigated by Merle<sup>3</sup> who proves the existence of blow-up solutions in the mass-critical regime and under some assumptions on  $K$  including in particular  $k_1 \leq K(x) \leq k_2$  with  $k_1, k_2$  being positive constants. Later on, the stability of standing waves was studied in Refs. 7 and 8 for  $K(x) = K(|x|)$  with  $K \in C^2(\mathbb{R}^N) \cap L^\infty(\mathbb{R}^N)$ ,  $\epsilon > 0$  small, and  $p \geq 1 + \frac{2}{N}$ . Recently, the Cauchy problem

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